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Ground-state symmetry classification for a non-isolated tunnelling particle

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Abstract. We demonstrate that the ground energy of a two-level system coupled with a bosonic environment can be labelled with a quantum number related to the total excitation number operator and independent of the coupling strength. Our approach is exact and is based on operator methods combining symmetry considerations with general properties of the lowest energy state. The relevance of our result in connection with the nature of the transition from the weak to strong coupling regime is briefly discussed.

The influence of environmental modes on tunnelling systems is ubiquitous in physics and chemistry. In the context of the fundamental problem of 'molecular structure' several authors have investigated the effect of the environment as a possible source of symmetry breaking [1]. It is, in fact, well known that certain molecules, exhibiting symmetric configurations, are localized in one of these configurations, instead of being delocalized as the discrete symmetry of the isolated molecule Hamiltonian would require [2, 3]. For example, this is the case for chiral molecules [1, 2, 4, 5].

The coupling of such molecules with the coordinates of the gas or condensed phase in which they are embedded may greatly reduce the energy splitting between the two delocalized symmetric configurations, leading in this way to localized eigenstates. Thus the stability of the localized or delocalized states is closely related to the existence of degeneracy in the ground state of the Hamiltonian that models the system.

These features are shared by the great variety of systems which can be represented as the interaction of a tunnelling unit, described by an effective one-dimensional symmetric double-well potential, with its surrounding medium. Ignoring the microscopic details of the particular physical system under consideration, it is possible to describe many of these physical [6], chemical [1, 2, 4, 7] and biological [5, 8] systems as a two-state particle interacting with a set of quantum harmonic oscillators. The correspondent Hamiltonian model offers a good conceptual starting point to simulate the effects of the environment on the dynamics of a 'small object' [4, 6, 7] and thus it turns out to be very useful in a variety of contexts such as paraelectric [9] or paramagnetic [10] defects in solids, tunnelling centres in metallic systems [11] or a two-level atom interacting with a quantized electromagnetic field [12]. Solving the eigenvalue problem posed by this Hamiltonian is unfortunately very difficult. Approximate approaches, such as variational [13], perturbative [14, 15] or variational–perturbative [2, 16], have therefore been worked out to provide reasonable analytical solutions. A fundamental unsolved issue regarding this system concerns the

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nature of its ground state when the values of the microscopic parameters do not make any perturbative approach legitimate [17]. In this paper we investigate the possibility of assigning a parameter-independent symmetry character to the lowest energy level of the system. It has been shown that, when the two-state particle is coupled to one quantum harmonic oscillator only, the lowest energy of the composite system is classifiable in terms of a quantum number related to a symmetry property of the interaction and independent of the coupling strength [15]. We will give a detailed and rigorous proof that such a result maintains its validity even when the more complex many-mode problem is investigated.

Our model consists of one non-isolated two-level particle (spin or pseudospin) interacting with N bosonic modes vis a dipole-like coupling. The correspondent Hamiltonian model is

$$H = \sum_{i=1}^{N} \hbar\omega_i \alpha_i^\dagger \alpha_i + \sum_{i=1}^{N} \varepsilon_i (\alpha_i + \alpha_i^\dagger)\sigma_x + \frac{\hbar\omega_0}{2}\sigma_z. \quad (1)$$

The two-state unit is characterized by an energy separation $\hbar\omega_0$ and described by Pauli operators σ_j, $(j = x, y, z)$. The ith oscillator has a frequency ω_i and its quanta are created or annihilated by the operators $\alpha_i^\dagger$ and α_i, respectively, satisfying the canonical commutation relations

$$[\alpha_i, \alpha_{i'}] = 0 \qquad [\alpha_i, \alpha_{i'}^\dagger] = \delta_{ii'}. \quad (2)$$

The microscopic positive parameter ε_i measures the coupling strength between the pseudospin and the ith mode of its bosonic environment.

It is immediately verified that the canonical transformation which changes the sign of α_i, $\alpha_i^\dagger$, σ_x, σ_y, leaving σ_z unmodified, is a symmetry of H [18, 19]. It is easy to convince oneself that this transformation can be accomplished by the following Hermitian operator:

$$P = \exp\left[\mathrm{i}\pi\left(\sum_{i=1}^{N}\alpha_i^\dagger\alpha_i + \frac{\sigma_z}{2} + \frac{1}{2}\right)\right]. \quad (3)$$

The operator P is thus a constant of motion for our problem. Moreover, it commutes with the total excitation number operator $\sum_{i=1}^{N}\alpha_i^\dagger\alpha_i + (\sigma_z/2) + (1/2)$ assuming the eigenvalue $+1$ (-1) in correspondence to even (odd) eigenvalues of such an operator. For this reason we refer to P simply as the parity operator. We denote by S_w the infinite-dimensional subspace of all the eigenstates of P with eigenvalue w.

Exploiting the symmetry property of the Hamiltonian model (1) expressed by $[H, P] = 0$ [16], we can reduce exactly the problem of its diagonalization to that of an effective Hamiltonian operator containing only (new) bosonic variables. Taking advantage of a treatment recently applied [19] to the Hamiltonian (1), we define the unitary operator

$$T = \exp\left\{-\mathrm{i}\frac{\pi}{2}\left[(\sigma_x - 1)\sum_{i=1}^{N}\alpha_i^\dagger\alpha_i\right]\right\}. \quad (4)$$

This operator transforms H into $\tilde{H} = T^\dagger H T$ as follows,

$$\tilde{H} = \sum_{i=1}^{N}\{\hbar\omega_i\alpha_i^\dagger\alpha_i + \varepsilon_i(\alpha_i^\dagger + \alpha_i)\} + \frac{\hbar\omega_0}{2}\sigma_z\prod_{i=1}^{N}\cos(\pi\alpha_i^\dagger\alpha_i) \quad (5)$$

and transforms P as

$$T^\dagger P T = -\sigma_z. \quad (6)$$

Since $[\tilde{H}, \sigma_z] = 0$, we may formally get rid of the operator σ_z in (5) regarding it as a c-number w equal to anyone of its eigenvalues. Thus the search of the common eigenvalues of H and P appears to be equivalent to the diagonalization of the following purely bosonic Hamiltonians ($w = \pm 1$):

$$H_w = \sum_{i=1}^{N} \{\hbar\omega_i \alpha_i^\dagger \alpha_i + \varepsilon_i(\alpha_i^\dagger + \alpha_i)\} - \frac{\hbar\omega_0}{2} w \prod_{i=1}^{N} \cos(\pi \alpha_i^\dagger \alpha_i). \tag{7}$$

In view of (6), in particular, by solving the eigenvalue problem of H_{+1} (H_{-1}) we may immediately build up all the exact eigenstates of H belonging to the invariant subspace of P corresponding to its positive (negative) eigenvalue.

It is well known [15] that the ground-state energy E_0 of a system may be recovered by taking the zero temperature limit of its free energy. Noting that for a physical system with Hamiltonian $\mathcal{H}$, in thermal equilibrium at temperature $T = 1/k\beta$ (k is the Boltzmann constant), the free energy F is defined as

$$F = -1/\beta \ln Z \tag{8}$$

where the partition function Z is given by

$$Z = \mathrm{Tr}[\exp(-\beta\mathcal{H})] \tag{9}$$

we may indeed write that (convergence is assumed)

$$E_0 = -\lim_{T\to 0} F = -\lim_{\beta\to\infty} \left\{ \frac{1}{\beta} \ln[Z] \right\}. \tag{10}$$

In general it is a formidable task to show the convergence of the trace of the operator $\exp(-\beta\mathcal{H})$ and of the free energy for $T \to 0$, in particular when, as in our problem, we do not know even approximately the solution of the relative eigenvalue problem. Since we wish to solve our problem on the basis of the representation of the lowest energy level as the zero temperature limit of F, we tacitly assume (as is usually done) that such a representation is meaningful whenever we need it during our demonstration.

We wish to apply equation (10) to the effective bosonic Hamiltonians (7), assuming the existence of a ground state for both H_{+1} and H_{-1}. Denoting by Z^+ (Z^-) the partition function relative to H_{+1} (H_{-1}) (and with Z^w the one relative to H_w), it is immediate to verify that the difference between the fundamental level of H_{+1}, E_g^+, and that of H_{-1}, here denoted by E_g^-, may be written as

$$E_g^+ - E_g^- = -\lim_{\beta\to\infty} \frac{1}{\beta} \ln\left[\frac{Z^+}{Z^-}\right]. \tag{11}$$

In order to evalute Z^w, we introduce appropriate non-normalized density operators relative to the N-mode problem in the form

$$\rho_w^{(N)}(\beta) = \exp(\beta H_w) = \exp(-\beta(H_0 + H_1)) \tag{12}$$

with

$$H_0 = \sum_{i=1}^{N} \{\hbar\omega_i \alpha_i^\dagger \alpha_i + \varepsilon_i(\alpha_i + \alpha_i^\dagger)\} \tag{13}$$

$$H_1 = -w \frac{\hbar\omega_0}{2} \prod_{i=1}^{N} \cos(\pi \alpha_i^\dagger \alpha_i). \tag{14}$$

We may use the Dyson expansion of the operator $\rho_w^{(N)}(\beta)$ [20], that is written identifying, in a formal sense only, H_1 as a 'small correction to H_0', but maintaining all the infinitely many terms in the series expansion because, in general, it is not legitimate to consider such a series as a perturbative expansion of the operator $\exp(-\beta(H_0 + H_1))$. Denoting by

$\rho_0^{(N)}(\beta)$ the non-normalized density operator relative to H_0, we may thus write the following identity:

$$\rho_w^{(N)}(\beta) = \rho_0^{(N)}(\beta) - \int_0^\beta \mathrm{d}\beta' \rho_0^{(N)}(\beta - \beta') H_1 \rho_0^{(N)}(\beta') + \int_0^\beta \mathrm{d}\beta' \int_0^{\beta'} \mathrm{d}\beta'' \{\rho_0^{(N)}(\beta - \beta') H_1 \rho_0^{(N)}(\beta' - \beta'') H_1 \rho_0^{(N)}(\beta'') - \cdots . \qquad (15)$$

To get the trace of $\rho_w^{(N)}(\beta)$, we seek an explicit expression for the trace of the $(n+1)$th term appearing on the right-hand side of equation (15):

$$(-1)^n \int_0^\beta \mathrm{d}\beta' \int_0^{\beta'} \mathrm{d}\beta'' \ldots \int_0^{\beta^{(n-1)}} \mathrm{d}\beta^{(n)} \operatorname{Tr}[\rho_0^{(N)}(\beta - \beta') H_1 \rho_0^{(N)}(\beta' - \beta'') \times H_1 \ldots \rho_0^{(N)}(\beta^{(n)})]. \qquad (16)$$

Exploiting the fact that H_1 is expressed as a product of single-mode operators and using a suitable complete set of vectors in the vector space V where H_{+1} and H_{-1} are defined, the trace of the complex N-mode operator appearing in equation (16) may be exactly transformed into a product of N traces of single-mode operators, all having the same mathematical structure. To prove this assertion we transform expression (16) as follows:

$$\begin{aligned}
&\rho_0^{(N)}(\beta - \beta') H_1 \rho_0^{(N)}(\beta' - \beta'') H_1 \ldots \rho_0^{(N)}(\beta^{(n)}) \\
&\quad = \prod_{i=1}^{N} \left\{ \exp[-(\beta - \beta')(\hbar\omega_i \alpha_i^\dagger \alpha_i + \varepsilon_i(\alpha_i + \alpha_i^\dagger))] \left(-w\frac{\hbar\omega_0}{2} \cos(\pi\alpha_i^\dagger\alpha_i) \right) \right. \\
&\quad \times \exp[-(\beta' - \beta'')(\hbar\omega_i \alpha_i^\dagger \alpha_i + \varepsilon_i(\alpha_i + \alpha_i^\dagger))] \left(-w\frac{\hbar\omega_0}{2} \cos(\pi\alpha_i^\dagger\alpha_i) \right) \\
&\quad \left. \ldots \left(-w\frac{\hbar\omega_0}{2} \cos(\pi\alpha_i^\dagger\alpha_i) \right) \exp[-\beta^{(n)}(\hbar\omega_i \alpha_i^\dagger \alpha_i + \varepsilon_i(\alpha_i + \alpha_i^\dagger))] \right\}. \qquad (17)
\end{aligned}$$

By choosing in V an arbitrary basis whose states are tensorial products of independent single-mode normalized states, we see from equation (17) that the determination of an explicit form of expression (16) is essentially reduced to the relatively simpler evaluation of the trace of the following single-mode operator:

$$[\rho_0(\beta - \beta') h_1 \rho_0(\beta' - \beta'') h_1 \ldots h_1 \rho_0(\beta^{(n)})] \qquad (18)$$

where

$$\rho_0(\beta) = \exp\{-\beta h_0\} \qquad (19)$$

and

$$h_0 = \hbar\omega\alpha^\dagger\alpha + \varepsilon(\alpha + \alpha^\dagger) \qquad (20)$$

$$h_1 = -w\frac{\hbar\omega_0}{2}\cos(\pi\alpha^\dagger\alpha). \qquad (21)$$

It is well known that the unitary displacement operator [14]

$$D(\gamma) = \exp[\gamma(\alpha^\dagger - \alpha)] \qquad (22)$$

accomplishes the canonical reduction of h_0 provided that we insert $\gamma = \varepsilon/\hbar\omega$. This circumstance, with the fact that $D(\gamma)$ (unless differently specified, we will henceforth write $D(\gamma)$ in place of $D(\varepsilon/\hbar\omega)$) acting on the vacuum state of the field mode generates a coherent state, makes the single-mode coherent basis a good choice for the evaluation of the trace of $\rho_w^{(1)}(\beta) \equiv \rho_w(\beta)$. It is easily seen that

$$D(\gamma)\rho_0(\beta)D^\dagger(\gamma) = \exp\left(\beta\frac{\varepsilon^2}{\hbar\omega}\right)\exp(-\beta\hbar\omega\alpha^\dagger\alpha) \tag{23}$$

from which we deduce that expression (18) may be written as follows:

$$\begin{aligned}[]&[\rho_0(\beta-\beta')h_1\rho_0(\beta'-\beta'')h_1\ldots h_1\rho_0(\beta^{(n)})]\\ &\quad= D^\dagger(\gamma)\exp(-(\beta-\beta')\hbar\omega\alpha^\dagger\alpha)\tilde{h}_1\exp(-(\beta'-\beta'')\hbar\omega\alpha^\dagger\alpha)\tilde{h}_1\ldots\\ &\qquad\times\tilde{h}_1\exp(-\beta^{(n)}\hbar\omega\alpha^\dagger\alpha)D(\gamma)\exp\left(\beta\frac{\varepsilon^2}{\hbar\omega}\right)\end{aligned} \tag{24}$$

where

$$\tilde{h}_1 = D(\gamma)h_1D^\dagger(\gamma). \tag{25}$$

Noting that

$$\alpha\cos(\pi\alpha^\dagger\alpha) = -\cos(\pi\alpha^\dagger\alpha)\alpha \tag{26}$$

it is not difficult to prove that h_1 satisfies the following property:

$$h_1D^\dagger(\gamma) = D(\gamma)h_1. \tag{27}$$

With the help of equation (27), the operator expressed by equation (24) becomes

$$\begin{aligned}[]&[\rho_0(\beta-\beta')h_1\rho_0(\beta'-\beta'')h_1\ldots h_1\rho_0(\beta^{(n)})]\\ &\quad= \left(-w\frac{\hbar\omega_0}{2}\right)^n\exp\left(\beta\frac{\varepsilon^2}{\hbar\omega}\right)\{D^\dagger(\gamma)\exp(-(\beta-\beta')\hbar\omega\alpha^\dagger\alpha)D(2\gamma)\\ &\qquad\times\exp(-(\beta'-\beta'')\hbar\omega\alpha^\dagger\alpha)D^\dagger(2\gamma)\ldots D^\dagger(2\gamma)\exp(-\beta^{(n)}\hbar\omega\alpha^\dagger\alpha)D(\gamma)\}\end{aligned} \tag{28}$$

if n is an even natural number, whereas it assumes the form

$$\begin{aligned}[]&[\rho_0(\beta-\beta')h_1\rho_0(\beta'-\beta'')h_1\ldots h_1\rho_0(\beta^{(n)})]\\ &\quad= \left(-w\frac{\hbar\omega_0}{2}\right)^n\exp\left(\beta\frac{\varepsilon^2}{\hbar\omega}\right)\{D^\dagger(\gamma)\exp(-(\beta-\beta')\hbar\omega\alpha^\dagger\alpha)\cos(\pi\alpha^\dagger\alpha)D^\dagger(2\gamma)\\ &\qquad\times\exp(-(\beta'-\beta'')\hbar\omega\alpha^\dagger\alpha)D(2\gamma)\ldots D^\dagger(2\gamma)\exp(-\beta^{(n)}\hbar\omega\alpha^\dagger\alpha)D(\gamma)\}\end{aligned} \tag{29}$$

when n is an odd natural number. Unfortunately, both operator expressions (28) and (29) still appear too involved for an estimate of their mean values on a single-mode coherent state. In order to simplify these expressions we should get rid of most of the operators $\exp(\beta^{(r)}\hbar\omega\alpha^\dagger\alpha)$ in equations (28) and (29). We can use the well known tranformation property [21],

$$\exp(x\alpha^\dagger\alpha)F(\alpha,\alpha^\dagger)\exp(-x\alpha^\dagger\alpha) = F(\alpha\,\mathrm{e}^{-x},\alpha^\dagger\,\mathrm{e}^{x}) \tag{30}$$

with $x \in C$, which permits us to put expressions (28) and (29) into the more convenient form

$$\begin{aligned}&\left(-w\frac{\hbar\omega_0}{2}\right)^n\exp\left(\beta\frac{\varepsilon^2}{\hbar\omega}\right)\\ &\qquad\times\{D^\dagger(\gamma)\exp(-\lambda\alpha^\dagger\alpha)\exp(a_1\alpha^\dagger-b_1\alpha)\ldots\exp(a_n\alpha^\dagger-b_n\alpha)D(\gamma)\}\end{aligned} \tag{31}$$

where $\{a_i\}$ and $\{b_i\}$ are real coefficients depending on $\beta', \beta'', \ldots, \beta^{(n)}$ and on the parameters present in the Hamiltonian model $h = h_0 + h_1$. Also, as far as the coefficient λ is concerned, we have $\lambda = \beta\hbar\omega$ if n is even and $\lambda = \beta\hbar\omega + \mathrm{i}\pi$ if n is odd.

The product of the n exponential operators $\prod_{i=1}^{n} \exp(a_i\alpha^\dagger - b_i\alpha)$ may be easily converted into a single exponential operator applying $(n-1)$ times the Glauber identity [22]. This way of proceeding yields the following result,

$$
\begin{aligned}
&[\rho_0(\beta-\beta')h_1\rho_0(\beta'-\beta'')h_1\ldots h_1\rho_0(\beta^{(n)})] \\
&\quad = \left(-w\frac{\hbar\omega_0}{2}\right)^n \exp\left(\beta\frac{\varepsilon^2}{\hbar\omega}\right) C_n(\beta',\beta'',\ldots,\beta^{(n)},\varepsilon,\hbar\omega) \\
&\qquad \times D^\dagger(\gamma)\exp(-\lambda\alpha^\dagger\alpha)\exp\left(2\frac{\varepsilon}{\hbar\omega}(f\alpha^\dagger - g\alpha)\right) D(\gamma)
\end{aligned}
\tag{32}
$$

where $C_n(\beta',\beta'',\ldots,\beta^{(n)},\varepsilon,\hbar\omega)$ is a positive c-number and f and g are real coefficients whose exact expressions are given as

$$
f = \sum_{k=1}^{n}(-1)^{k+n+1}\exp(\beta^{(k)}\hbar\omega) \tag{33}
$$

$$
g = \sum_{k=1}^{n}(-1)^{k+n+1}\exp(-\beta^{(k)}\hbar\omega). \tag{34}
$$

As a consequence of the operator identity expressed by equation (32), we can now evaluate the trace of the operator defined by equation (18) using the coherent-states basis. Writing the operator $\exp(2(\varepsilon/\hbar\omega)(f\alpha^\dagger - g\alpha))$ in its normal form and using once more the transformation property given in equation (30), we get

$$
\begin{aligned}
&\mathrm{Tr}\left[D^\dagger(\gamma)\exp(-\lambda\alpha^\dagger\alpha)\exp\left(2\frac{\varepsilon}{\hbar\omega}(f\alpha^\dagger - g\alpha)\right)D(\gamma)\right] \\
&\quad = \int \frac{\mathrm{d}^2\delta}{\pi}\langle\delta|D^\dagger(\gamma)\exp(-\lambda\alpha^\dagger\alpha)\exp\left(2\frac{\varepsilon}{\hbar\omega}(f\alpha^\dagger - g\alpha)\right)D(\gamma)|\delta\rangle \\
&\quad = \int \frac{\mathrm{d}^2\delta}{\pi}\exp(x+y\delta+z\delta^*)\left\langle\frac{\varepsilon}{\hbar\omega}+\delta\right|\exp(-\lambda\alpha^\dagger\alpha)\left|\frac{\varepsilon}{\hbar\omega}+\delta\right\rangle
\end{aligned}
\tag{35}
$$

where the integration domain is the whole complex plane. The real coefficients x, y, z depend in a complicated form on f, g and β and their explicit dependence is not relevant for our purposes. It is easy to evaluate the mean value appearing in the integrand in the last line of equating (35) obtaining [23]

$$
\begin{aligned}
&\int \frac{\mathrm{d}^2\delta}{\pi}\exp(x+y\delta+z\delta^*)\left\langle\frac{\varepsilon}{\hbar\omega}+\delta\right|\exp(-\lambda\alpha^\dagger\alpha)\left|\frac{\varepsilon}{\hbar\omega}+\delta\right\rangle \\
&\quad = \int \frac{\mathrm{d}^2\delta}{\pi}\exp(W+\bar{X}\delta+\bar{Y}\delta^*-\bar{Z}|\delta|^2) = \frac{1}{\bar{Z}}\exp(W)\exp\left(\frac{\bar{X}\bar{Y}}{\bar{Z}}\right)
\end{aligned}
\tag{36}
$$

where W, $\bar{X}$, $\bar{Y}$, $\bar{Z}$ are real functions of f, g and β. We give here, for example, the expression for $\bar{Z}$,

$$
\bar{Z} = 1 + (-1)^{n+1}\exp(-\beta\hbar\omega) \tag{37}
$$

which shows that $\bar{Z} > 0$ but, in contrast, we leave undefined the more intricate functions W, $\bar{X}$, $\bar{Y}$ as we did for x, y, z. Putting together equation (32) and (36) finally yields

$$
\begin{aligned}
&\mathrm{Tr}[\rho_0(\beta-\beta')h_1\rho_0(\beta'-\beta'')h_1\ldots h_1\rho_0(\beta^{(n)})] \\
&\quad = \left(-w\frac{\hbar\omega_0}{2}\right)^n \exp\left(\beta\frac{\varepsilon^2}{\hbar\omega}\right) F_n(\beta,\beta',\beta'',\ldots,\beta^{(n)},\varepsilon,\omega)
\end{aligned}
\tag{38}
$$

where the function $F_n(\beta, \beta', \beta'', \ldots, \beta^{(n)}, \varepsilon, \omega)$ is, by its construction, positive. Now reconsidering equation (17) and taking the trace of both sides, we may easily take advantage of the result expressed by equation (38), relative to a single-mode problem, in the N-mode correspondent problem, obtaining

$$\mathrm{Tr}[\rho_0^{(N)}(\beta-\beta')H_1\rho_0^{(N)}(\beta'-\beta'')H_1\ldots H_1\rho_0^{(N)}(\beta^{(n)})]$$
$$= \left(-w\frac{\hbar\omega_0}{2}\right)^n \exp\left(\beta\sum_{i=1}^{N}\frac{\varepsilon_i^2}{\hbar\omega_i}\right)\prod_{i=1}^{N}F_n(\beta, \beta', \beta'', \ldots, \beta^{(n)}, \varepsilon_i, \omega_i). \tag{39}$$

Evaluating the integrals with respect to $\beta', \beta'', \ldots, \beta^{(n)}$ according to definition (17), after a formal summation of the Dyson series (15), we finally achieve the following expression for the partition function Z^w:

$$Z^w = \exp\left(\beta\sum_{i=1}^{N}\frac{\varepsilon_i^2}{\hbar\omega_i}\right)\sum_{n=0}^{\infty}\left(w\frac{\hbar\omega_0}{2}\right)^n B_n(\beta, \{\varepsilon_i\}, \{\omega_i\}) \tag{40}$$

where $B_n(\beta, \{\varepsilon_i\}, \{\omega_i\})$ is positive for any n. Despite the insurmountable difficulty in getting explicit expressions for the coefficients $B_n(\beta, \{\varepsilon_i\}, \{\omega_i\})$ for every n, we can, however, succeed in drawing interesting physical conclusions based only on the knowledge of the sign of all these coefficients. Returning to equation (11), we, in fact, deduce that

$$E_{\mathrm{g}}^{+} - E_{\mathrm{g}}^{-} = -\lim_{\beta\to\infty}\left\{\frac{1}{\beta}\ln\left[\frac{\sum_{n=0}^{\infty}(\hbar\omega_0/2)^n B_n(\beta, \{\varepsilon_i\}, \{\omega_i\})}{\sum_{n=0}^{\infty}(-\hbar\omega_0/2)^n B_n(\beta, \{\varepsilon_i\}, \{\omega_i\})}\right]\right\} \tag{41}$$

and, since

$$\frac{\sum_{n=0}^{\infty}(\hbar\omega_0/2)^n B_n(\beta, \{\varepsilon_i\}, \{\omega_i\})}{\sum_{n=0}^{\infty}(-\hbar\omega_0/2)^n B_n(\beta, \{\varepsilon_i\}, \{\omega_i\})} > 1 \tag{42}$$

we arrive at the conclusion

$$E_{\mathrm{g}}^{+} - E_{\mathrm{g}}^{-} \leqslant 0. \tag{43}$$

Because $E_{\mathrm{g}} = \min(E_{\mathrm{g}}^{+}, E_{\mathrm{g}}^{-})$, we may elucidate the physical meaning of equation (43) by saying that, in consequence of its validity in the whole parameter space, a ground state of the interaction between the two-state particle and its environment simulated by N quantum oscillators always belongs to the positive-parity subspace of the total Hilbert space of the system. This amounts to excluding the possibility of crossing between the lowest energy eigenstates of H evaluated inside S_{+1} and S_{-1}, respectively. The absence of parity crossing in the ground state of the combined particle–bosonic environment interaction legitimates the search inside S_{+1} for a ground state of the Hamiltonian model (1).

We wish to conclude by adding some final remarks on the content of this paper. Our approach is based only on operator methods combining symmetry considerations and general properties of the ground state. A distinctive feature to point out is the general validity of the result that we have obtained. In fact, it has been deduced rigorously without assigning a specific dispersion law to the bosonic environment and without fixing a particular mode dependence of the coupling constants. This means that they are valid over the whole range of the characteristic parameters appearing in the Hamiltonian. Another interesting feature is the fact that the mathematical technique employed to achieve the result is quite independent from the resolution of the eigenvalue problem of H.

Our result is of relevance in the context of the problem concerning the nature of the transition connecting the nearly-free and the self-trapped regimes. Besides its theoretical intrinsic interest, this fact has experimental significance too, considering that it has been recently reported that the microscopic parameters of some systems represented by (1) can be tuned using macroscopic external parameters [24].

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